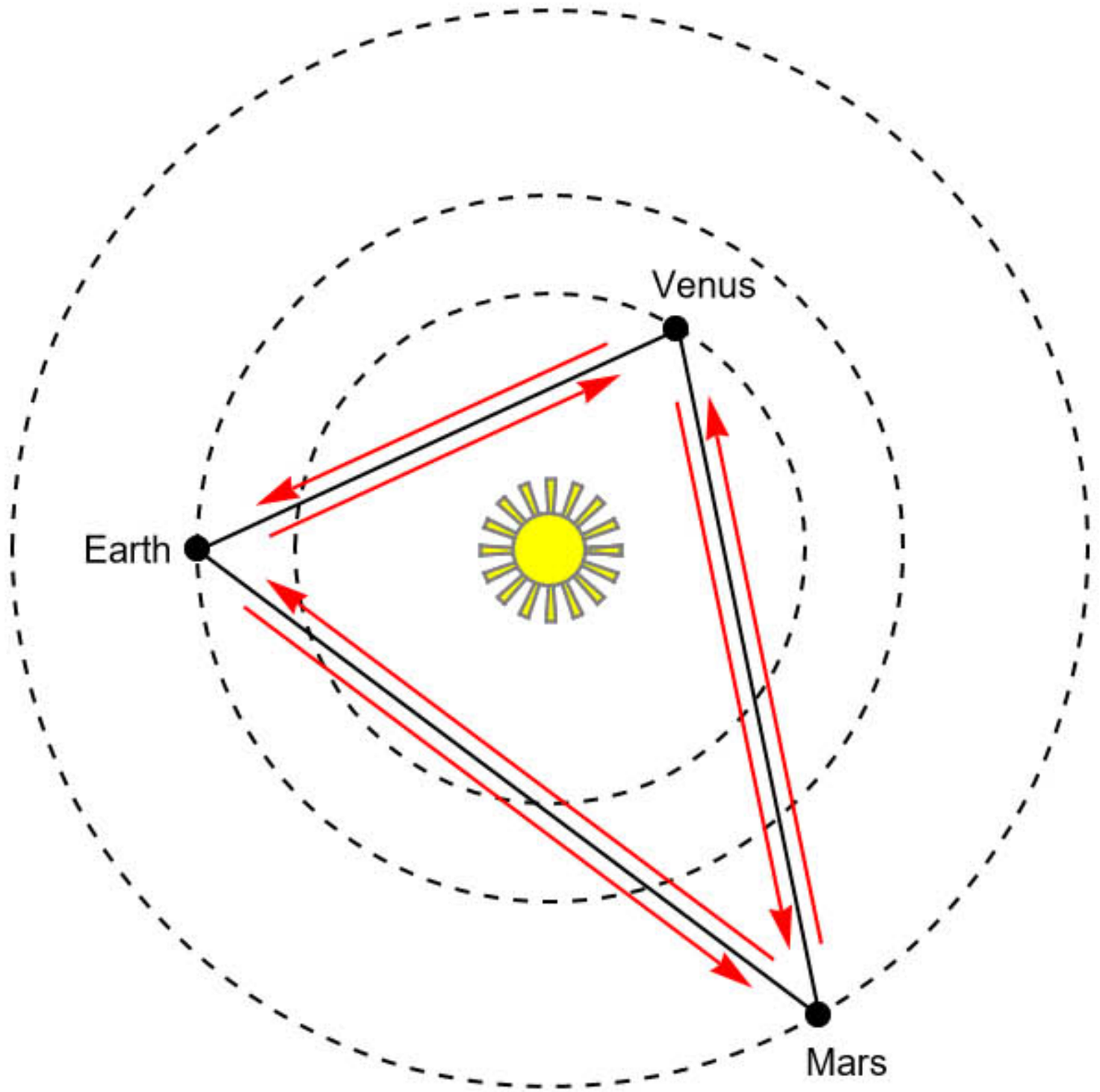
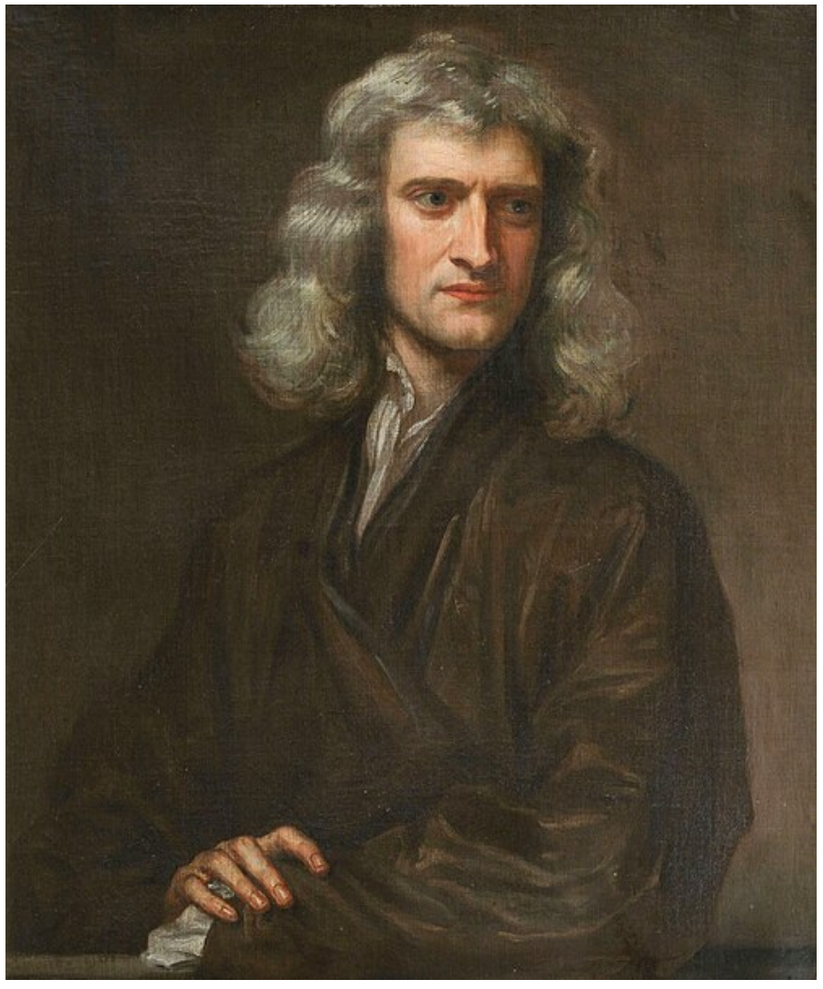

Venus
Earth
Mars

“An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, … would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past could be present before its eyes.”

— Pierre Simon Laplace (1814)

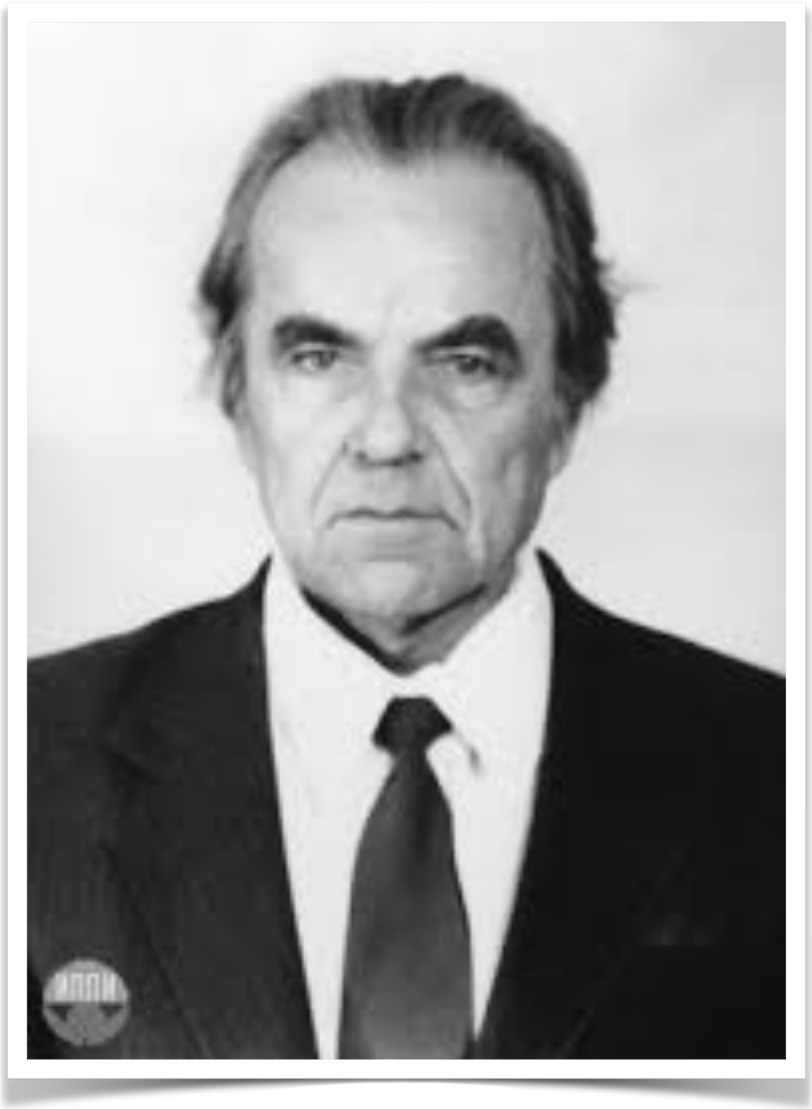
“For while comets move in very excentrick orbs in all manner of positions, blind fate could never make all the planets move one and the same way in orbs concentrick, some inconsiderable irregularities excepted, which may have risen from the mutual actions of comets and planets upon one another, and which will be apt to increase, till this system wants a reformation”

— Isaac Newton (1717)

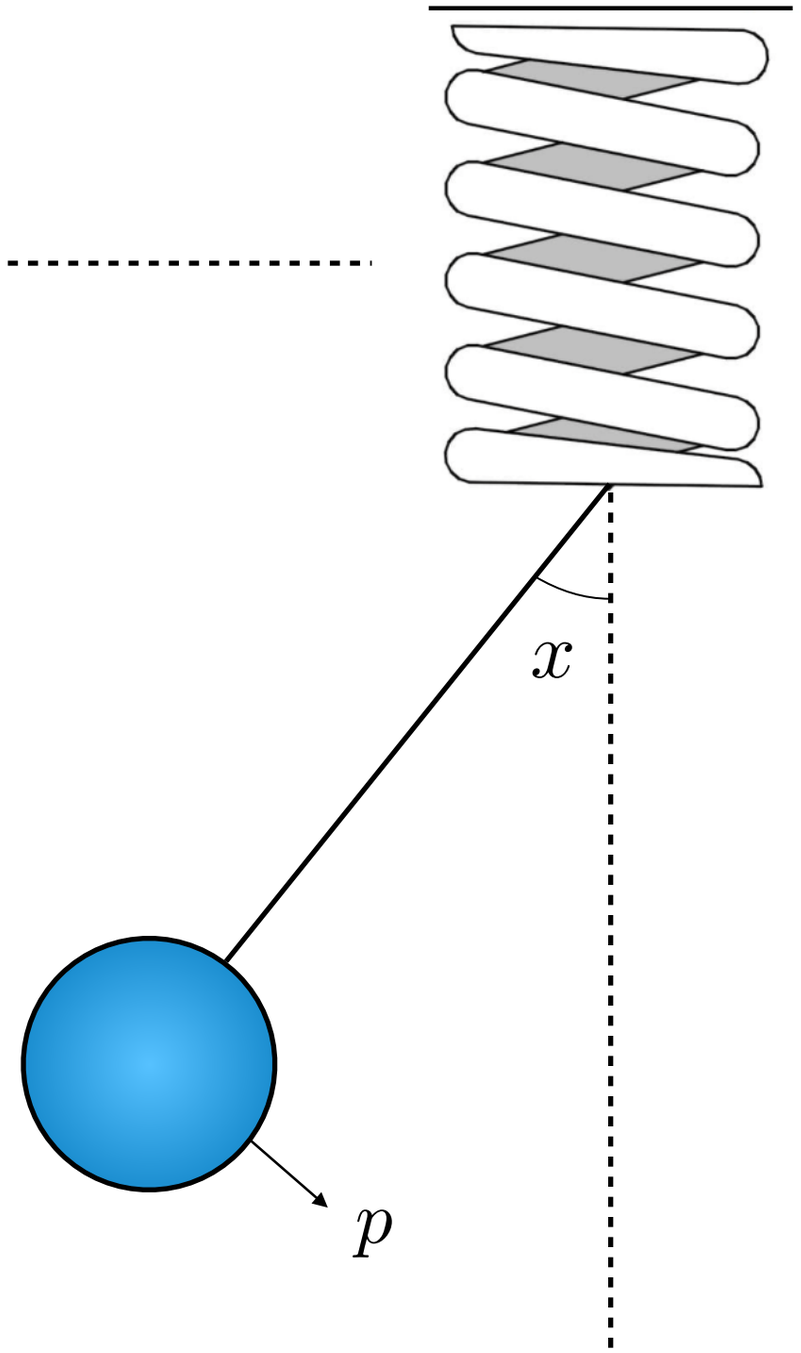


"Basic model is forced pendulum. It is elementary phenomenon of nonlinear dynamics, a cell, which forms all the diversity of nonlinear processes."

- B. V. Chirikov (1979)

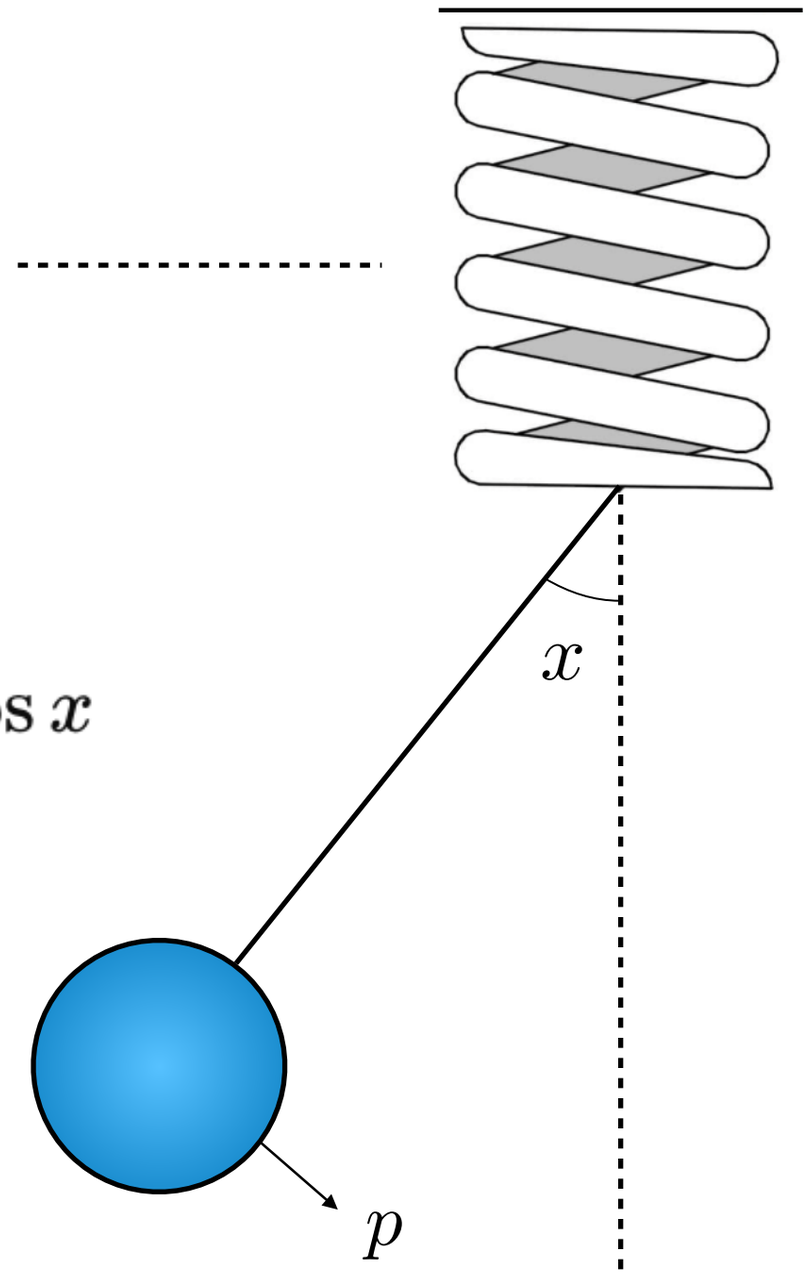


periodic forcing



periodic forcing

$$H(x,p,t) \;=\; \frac{p^2}{2} \;+\; c\,(1+\varepsilon\cos(\omega t))\,\cos x$$

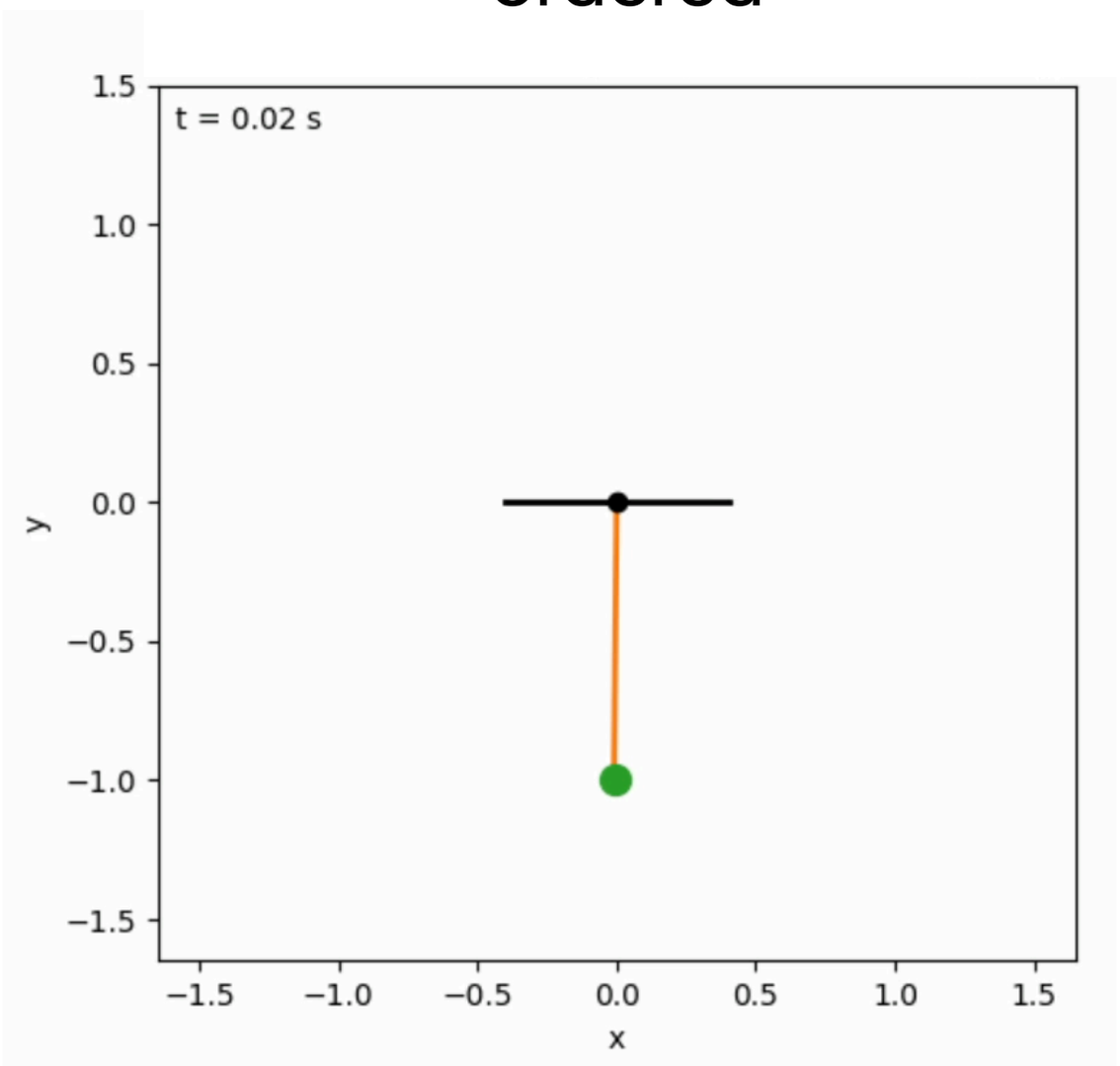


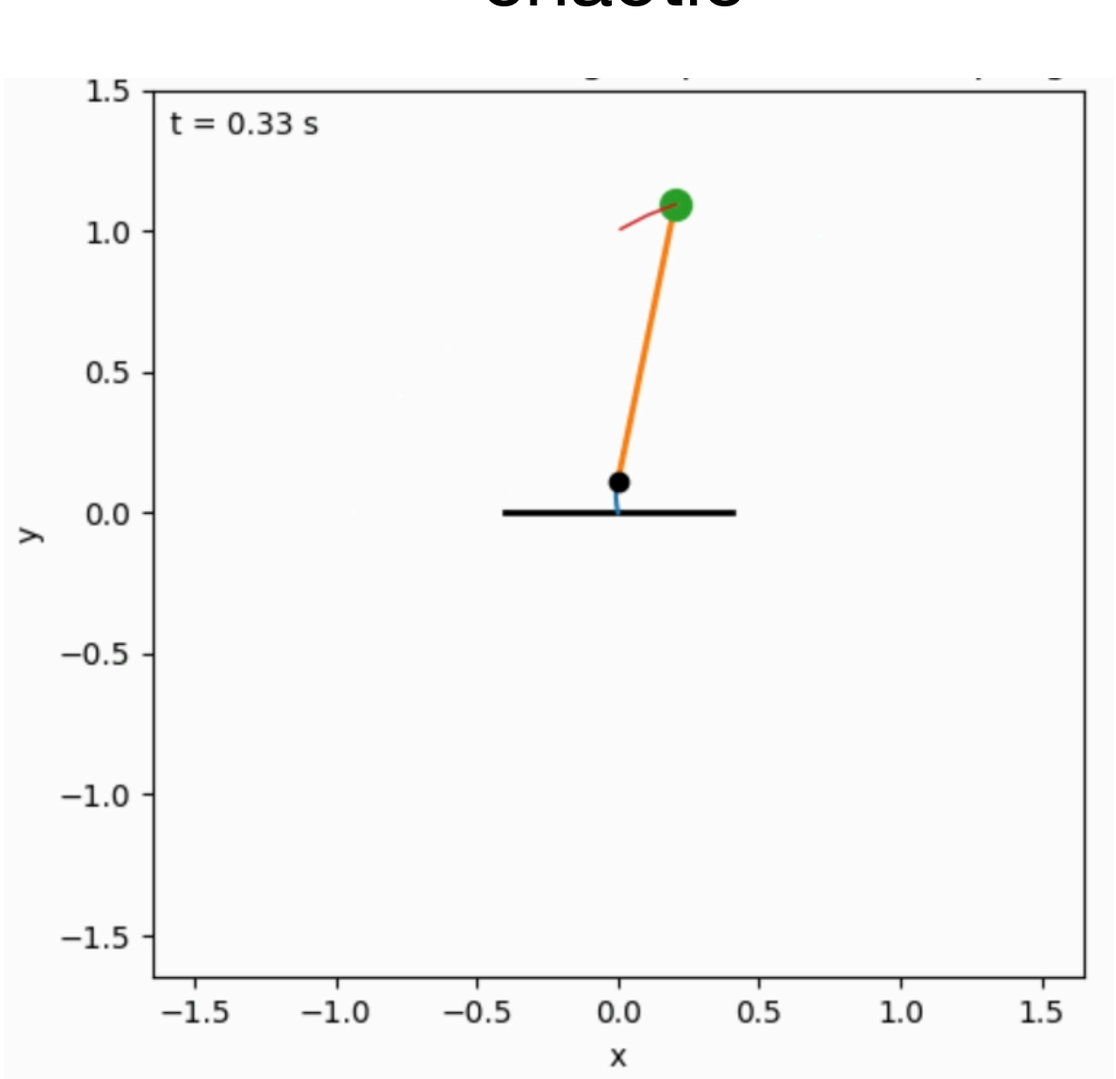
$$\begin{cases} \dot{x} = \dfrac{\partial H}{\partial p} = p \\ \dot{p} = -\dfrac{\partial H}{\partial x} = c\,(1+\varepsilon\cos(\omega t))\sin x \end{cases}$$

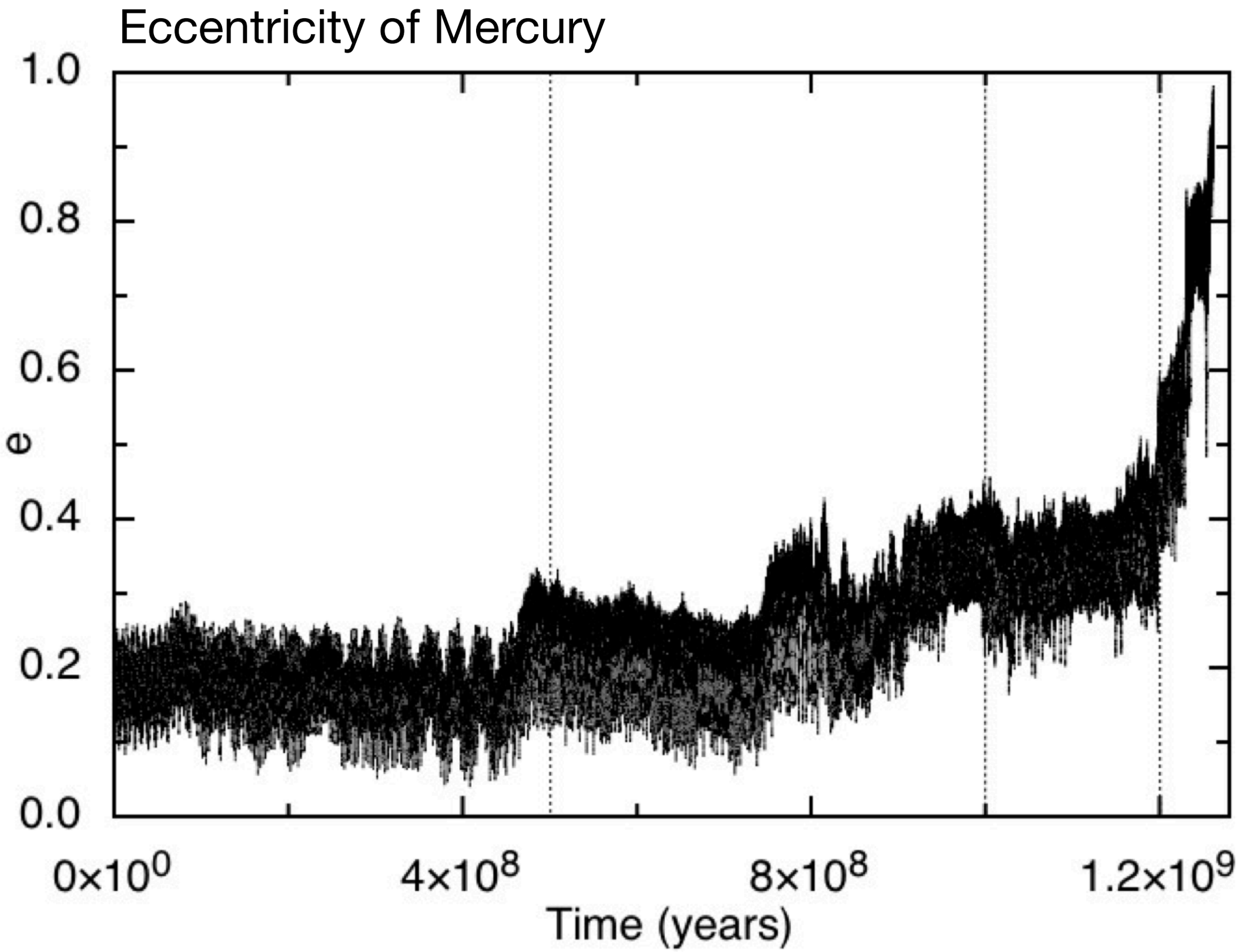
dynamics

ordered

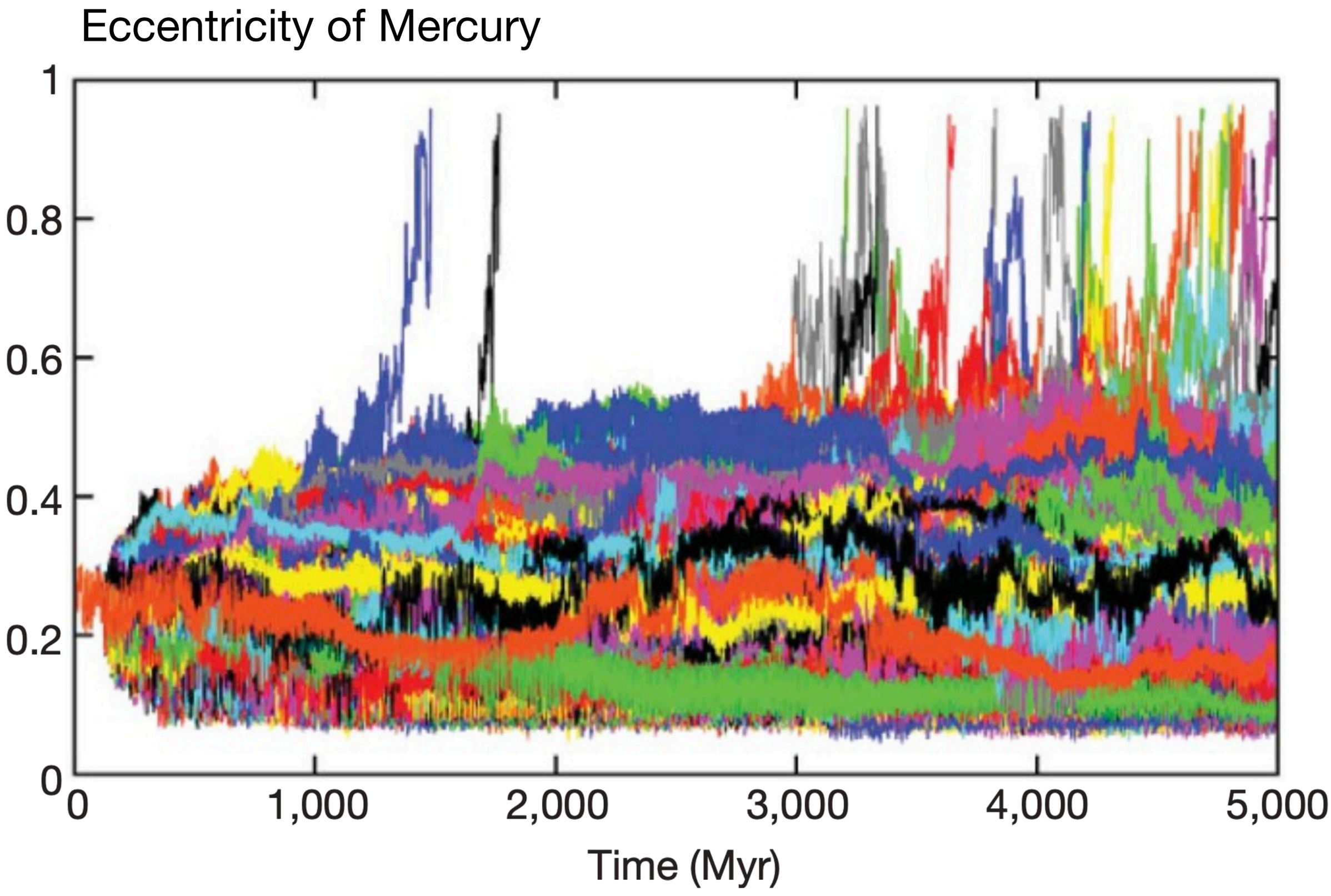
chaotic





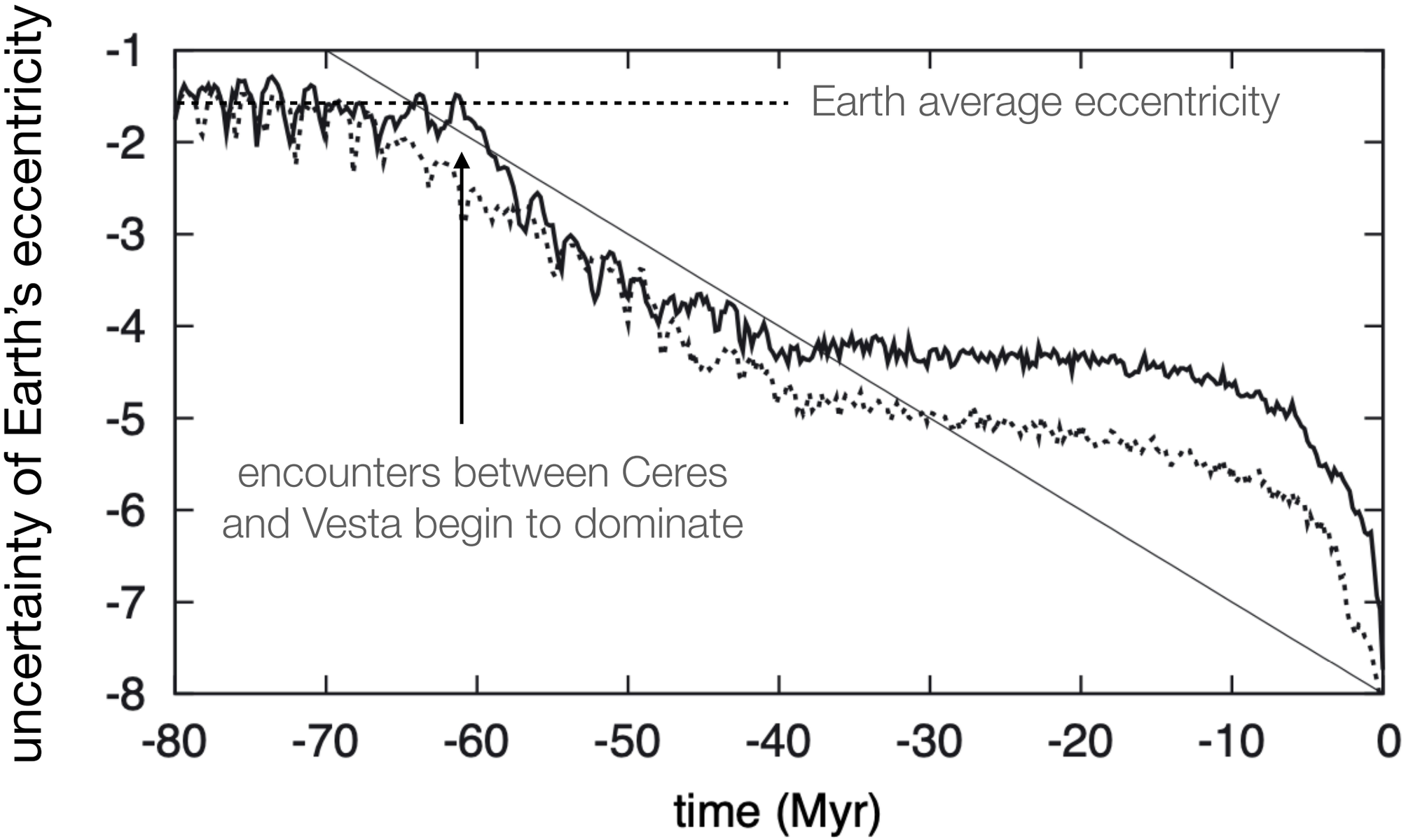


(Batygin & Laughlin, 2008)



(Laskar & Gastineau, 2008)

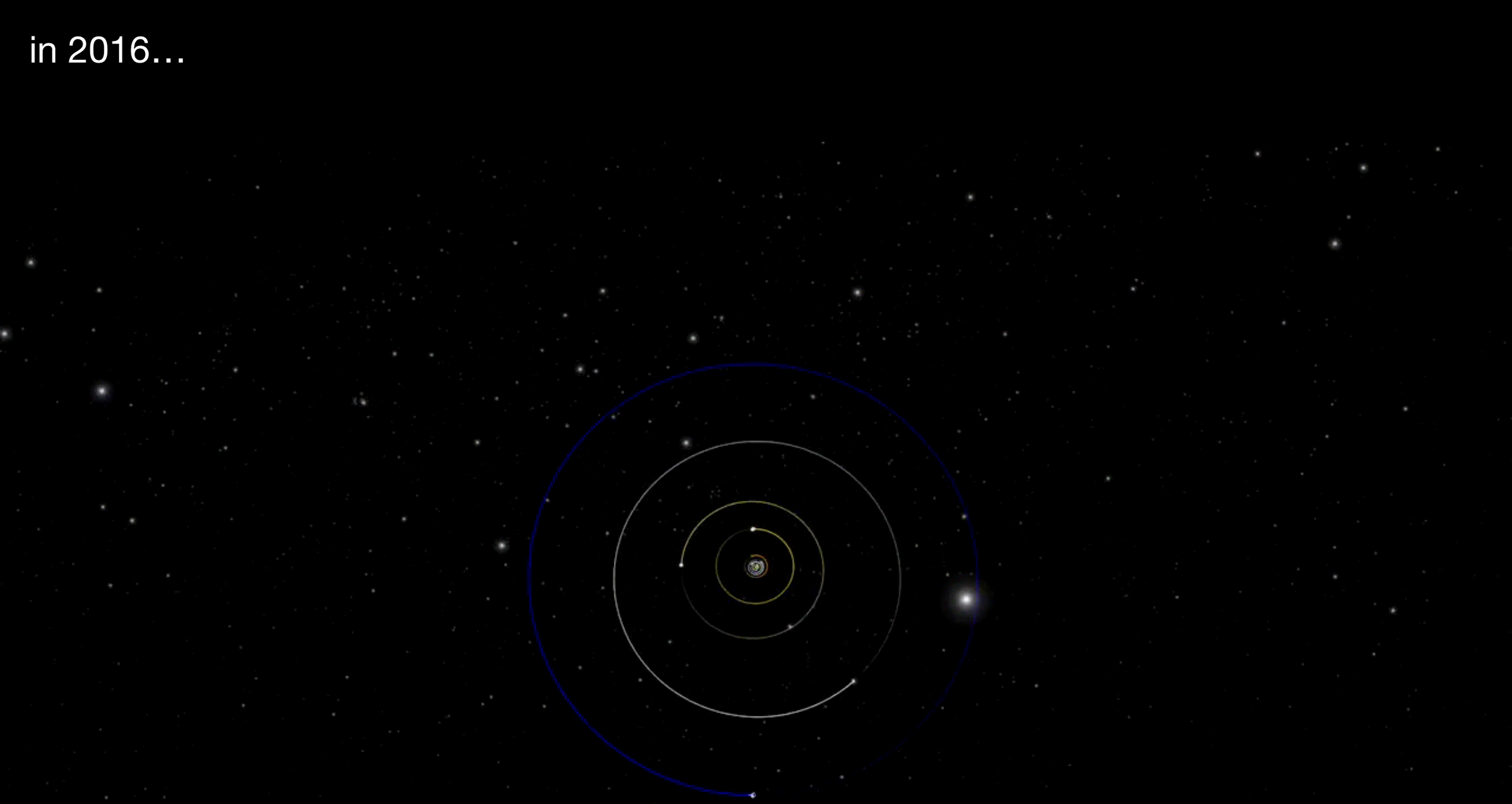
we will never know (from dynamics alone)



(Laskar et al., 2011)

Chaos biases towards short-timescale problems

in 2016...



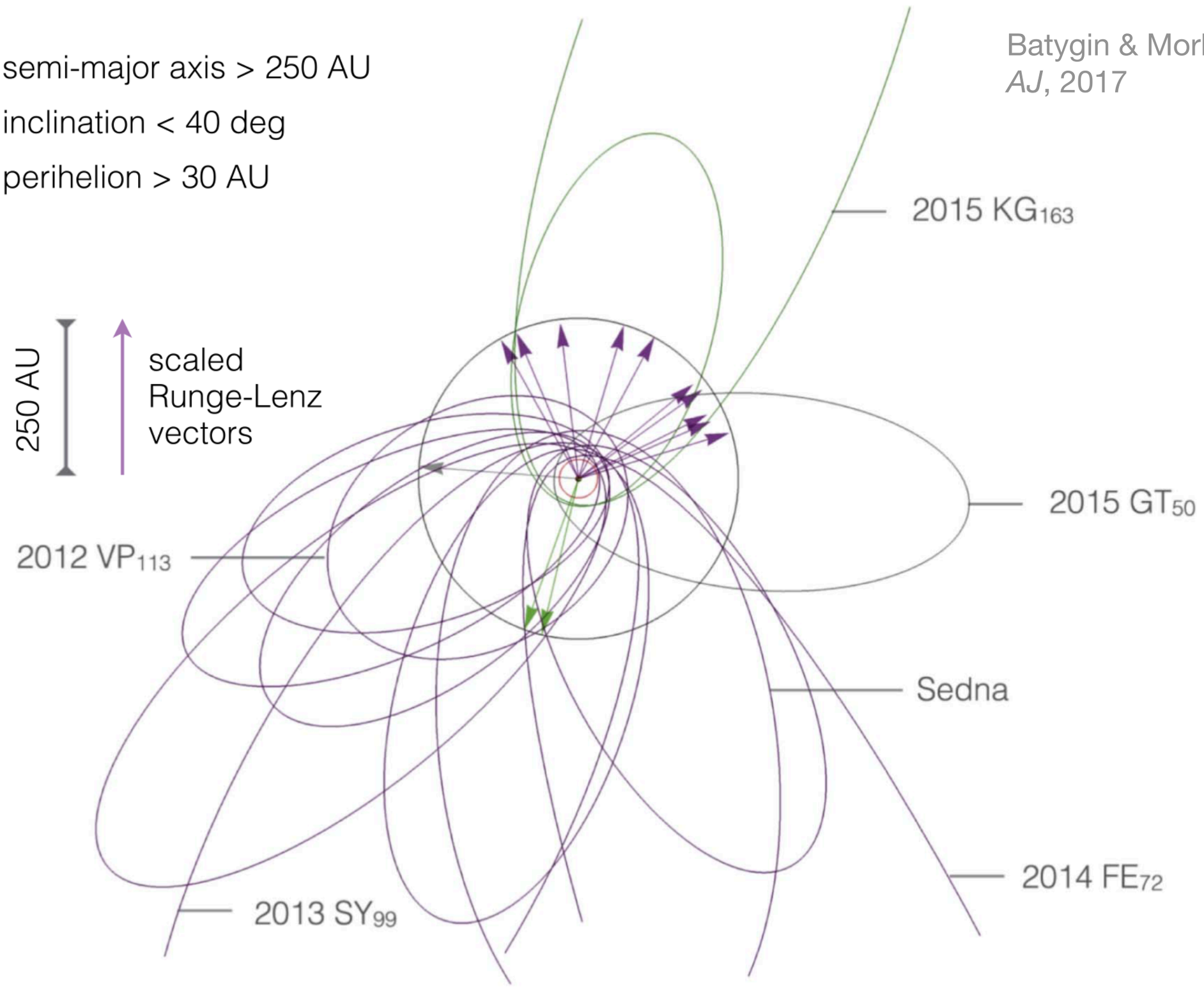
CLUSTERING

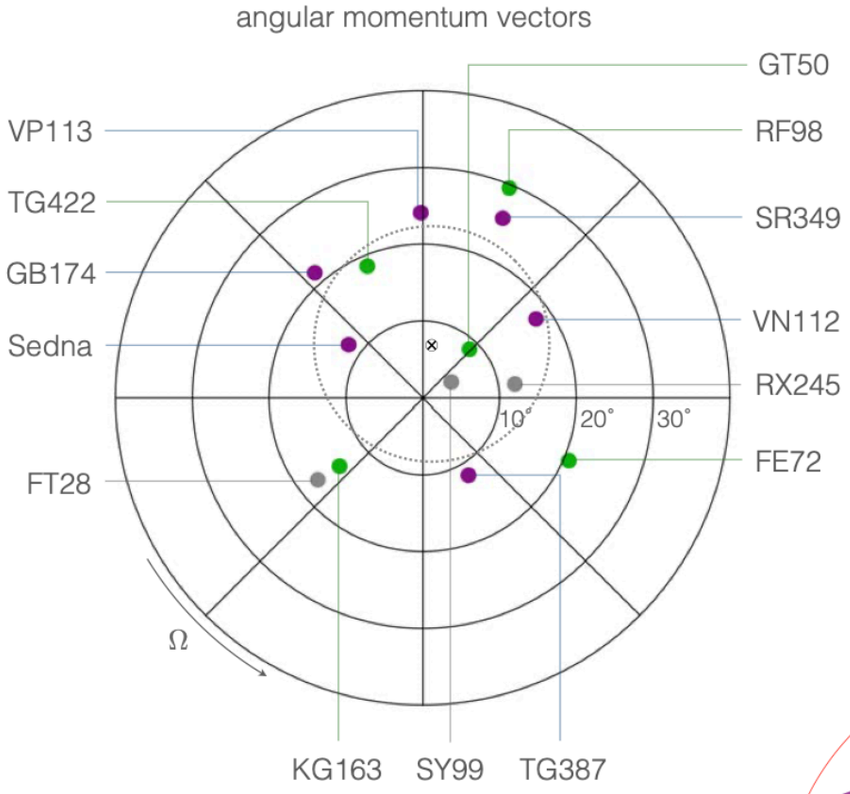
Batygin & Morbidelli,
AJ, 2017

semi-major axis > 250 AU

inclination < 40 deg

perihelion > 30 AU





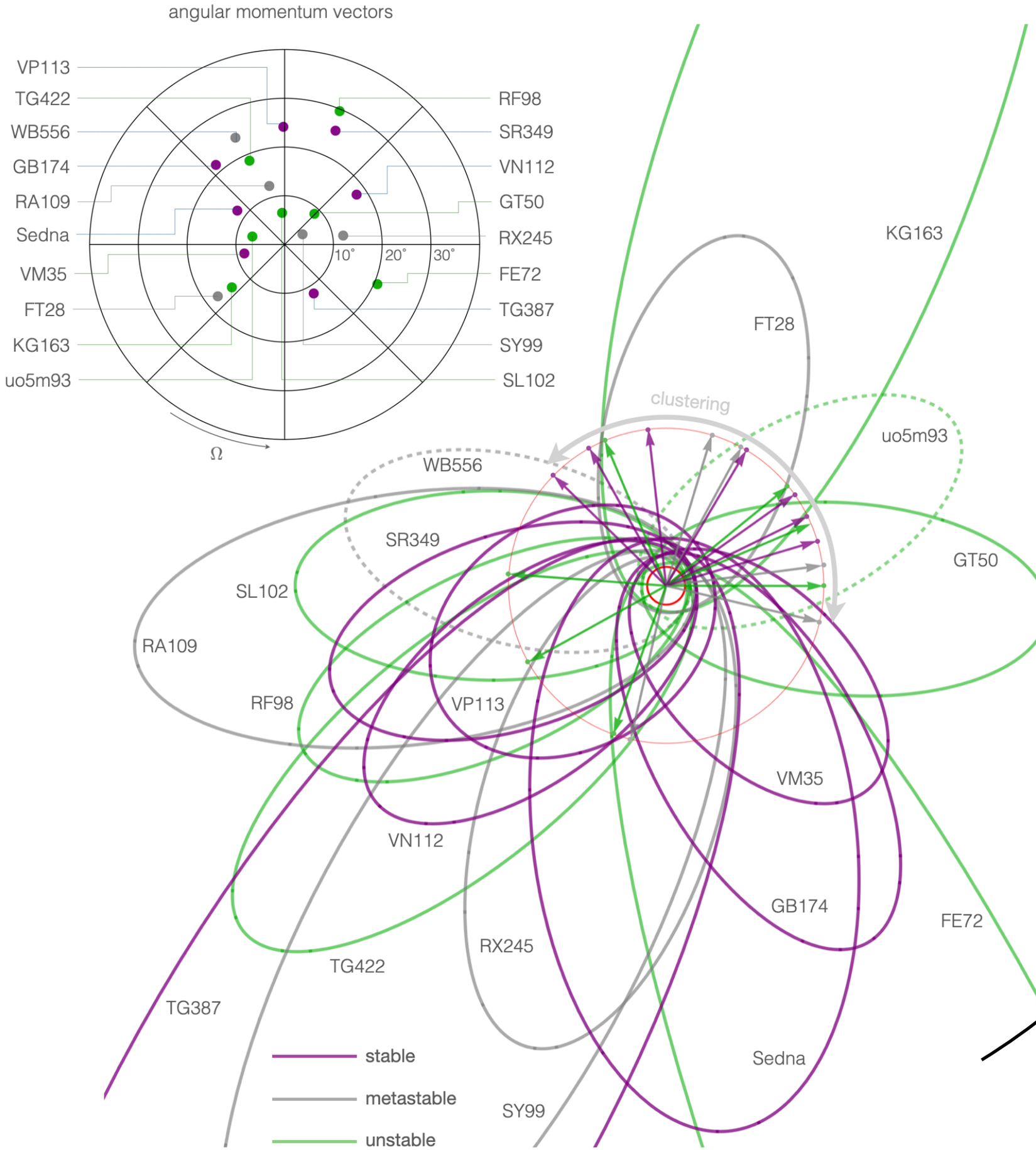
CLUSTERING

Batygin, Adams,
Brown, Becker, *Phys.R.*, 2019

stable

metastable

unstable


angular momentum vectors
VP113
TG422
WB556
GB174
RA109
Sedna
VM35
FT28
KG163
uo5m93
RF98
SR349
VN112
GT50
RX245
FE72
TG387
SY99
SL102
10°
20°
30°
Ω
CLUSTERING
Batygin & Brown,
ApJL, 2021
stable
metastable
unstable
clustering
250 AU
precession timescale:
a few x 100 Myr

Pichierri & Batygin, 2025 *in press*

KG163 FT28 RR205 SX403 RY245 WB556 SL102 VJ305 RA109 SA59 SR349 RF98 SD106 VP113 VN112 TG422 SG58 TU115 TG387 SY99 RX245 UT15 CR105 SEDNA FE72 GB174 BP519 VM35 CL54 FL28 GT50 FP185 DK18 DM319 GB32 EU5

Max Diffusion Coefficient [AU^2/yr]

10^{1} 10^{-1} 10^{-3} 10^{-5} 10^{-7} 10^{-9}

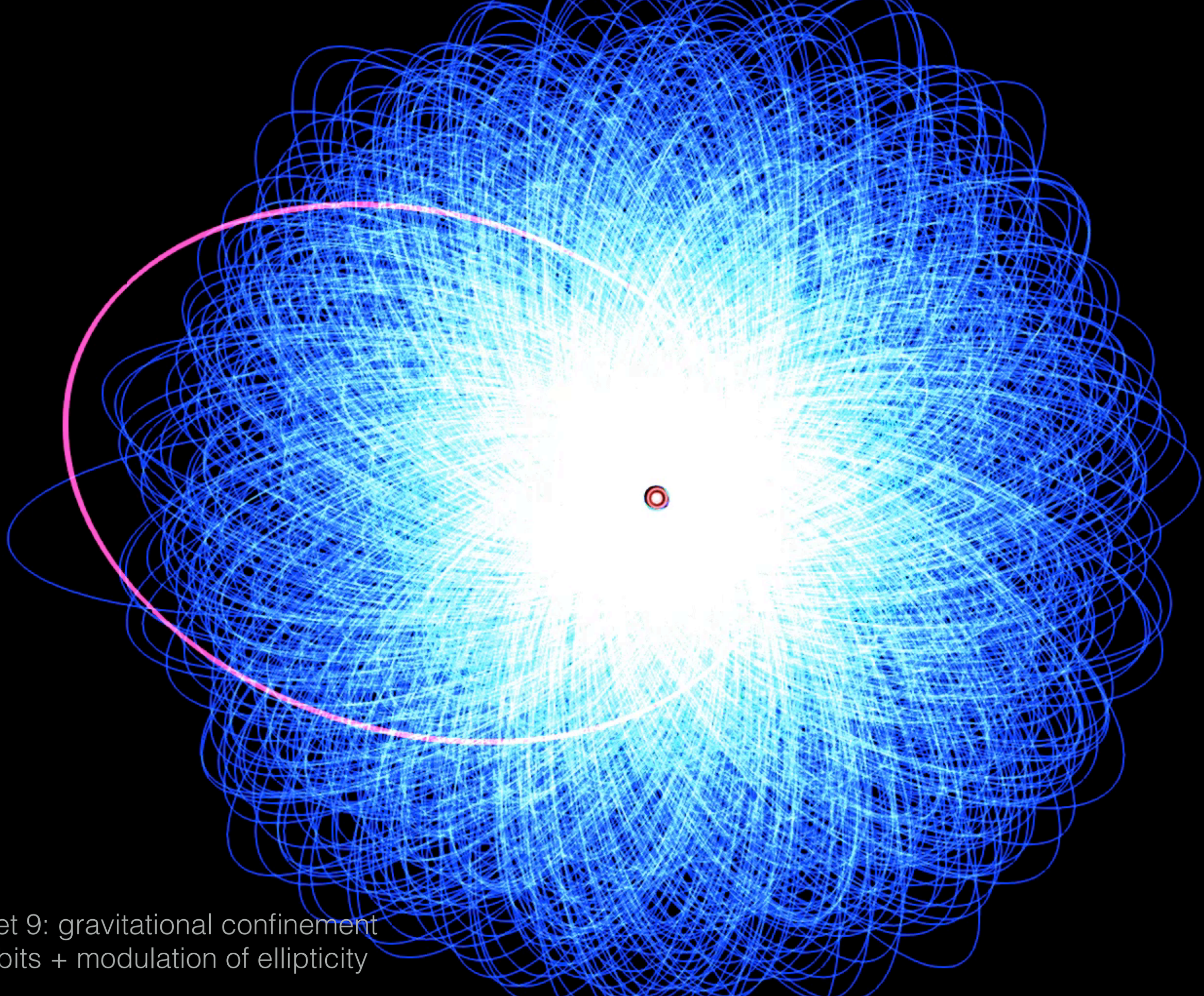
$\mathcal{D}_{a,\mathrm{crit}}$

-180 -135 -90 -45 0 45 90 135 180

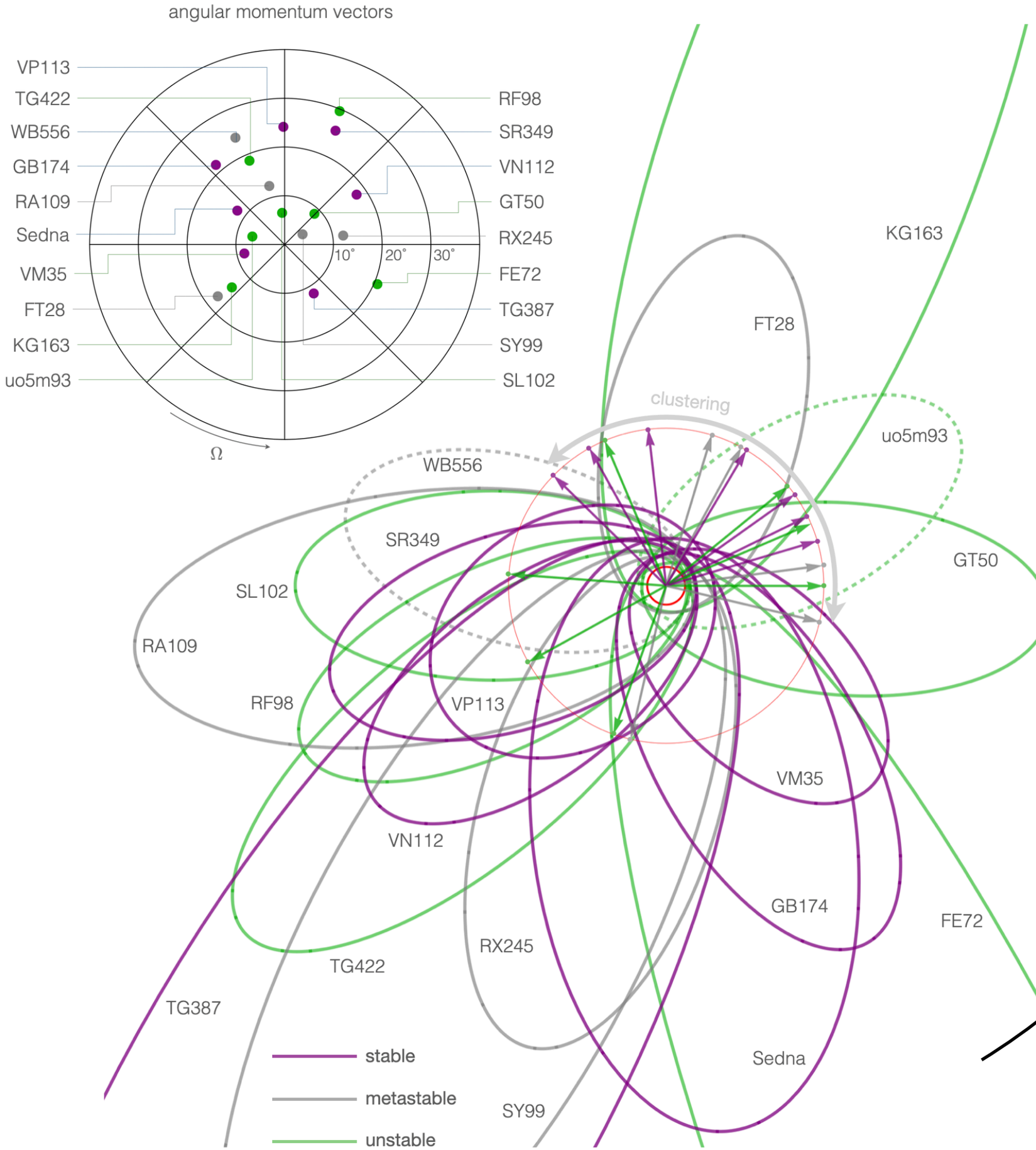
Clustering ($\varpi_{\mathrm{tp}} - \varpi^*$)

Stable
Metastable
Unstable

$a_{\mathrm{TNO}} > 200$ AU, $q_{\mathrm{init,TNO}} > 30$ AU



Planet 9: gravitational confinement
of orbits + modulation of ellipticity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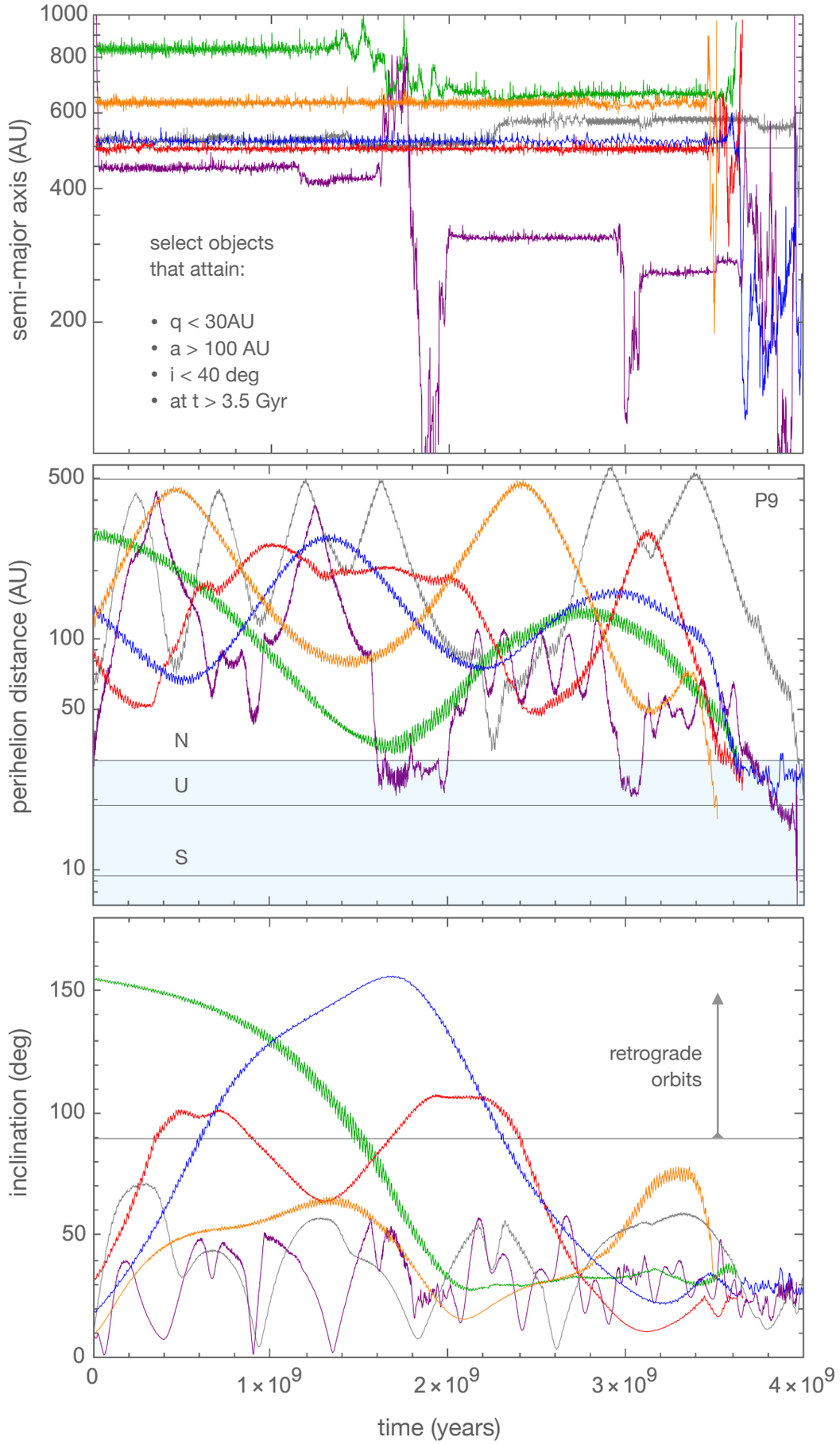
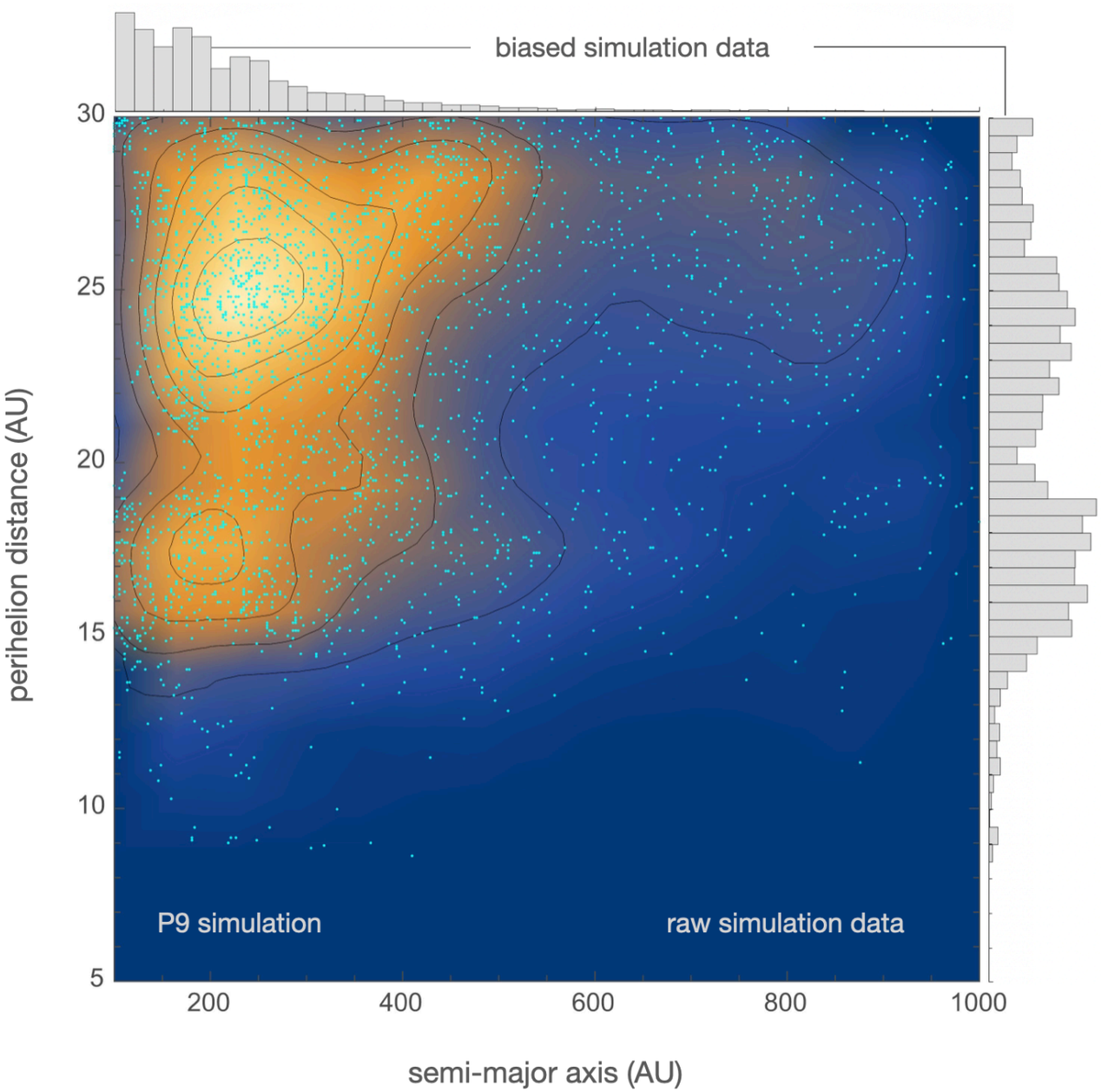
SL102
10°
20°
30°
Ω
CLUSTERING
Batygin & Brown,
ApJL, 2021
stable
metastable
unstable
clustering
250 AU
precession timescale:
a few x 100 Myr

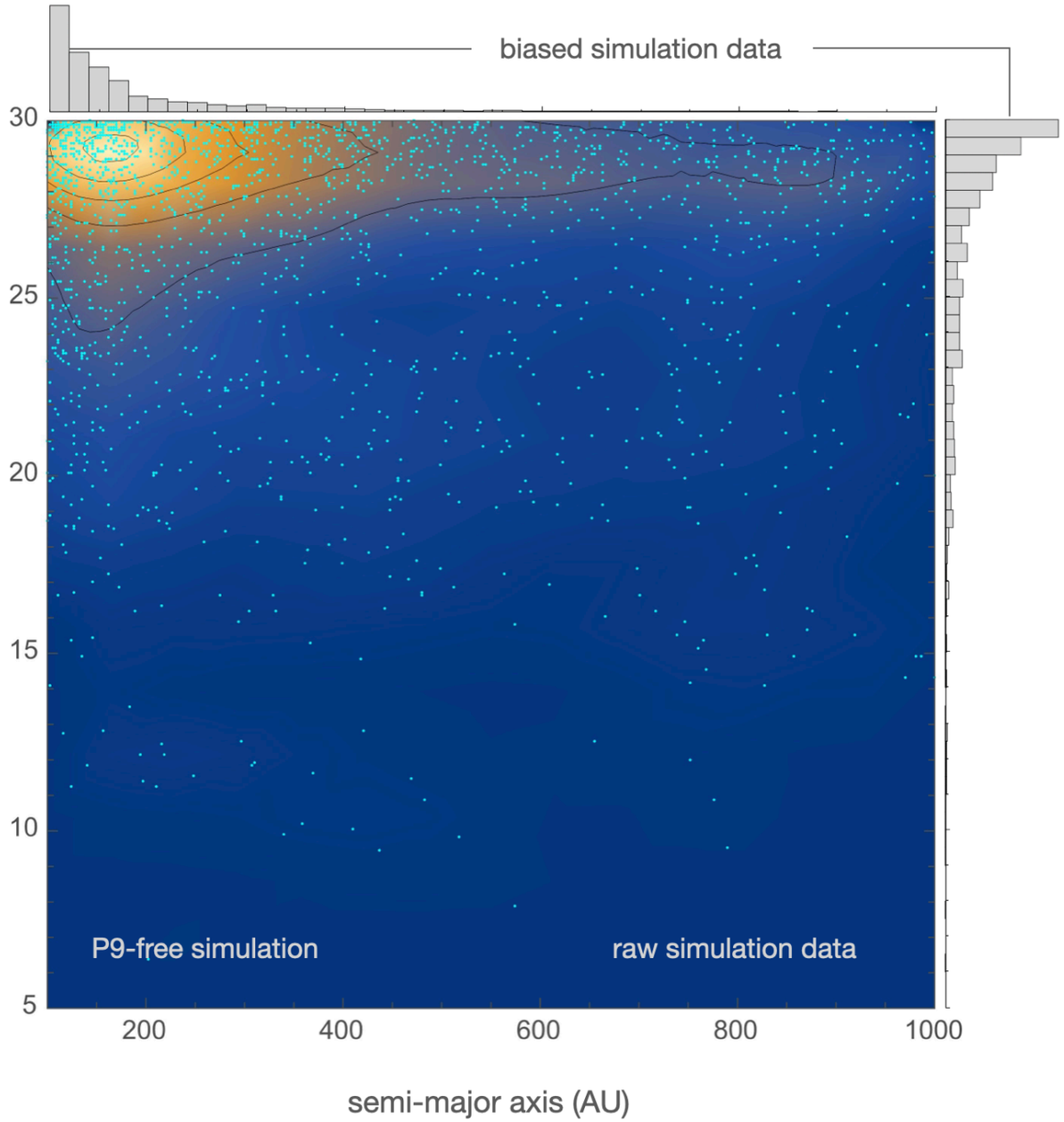
If Planet 9 exists, it should drive a steady flux of long-period Neptune-crossing objects

Batygin et al 2024, *Astrophysical Journal Letters*

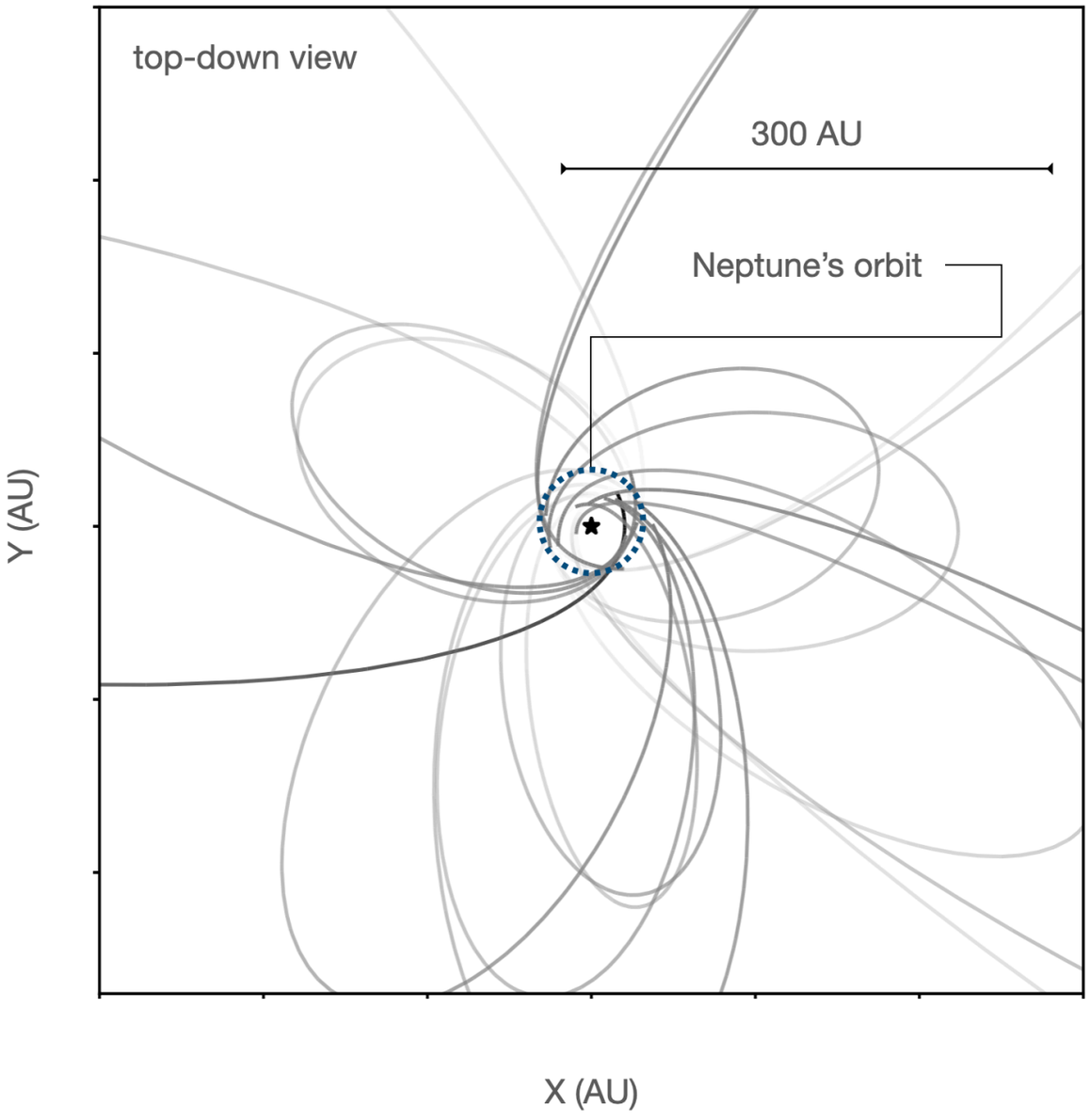


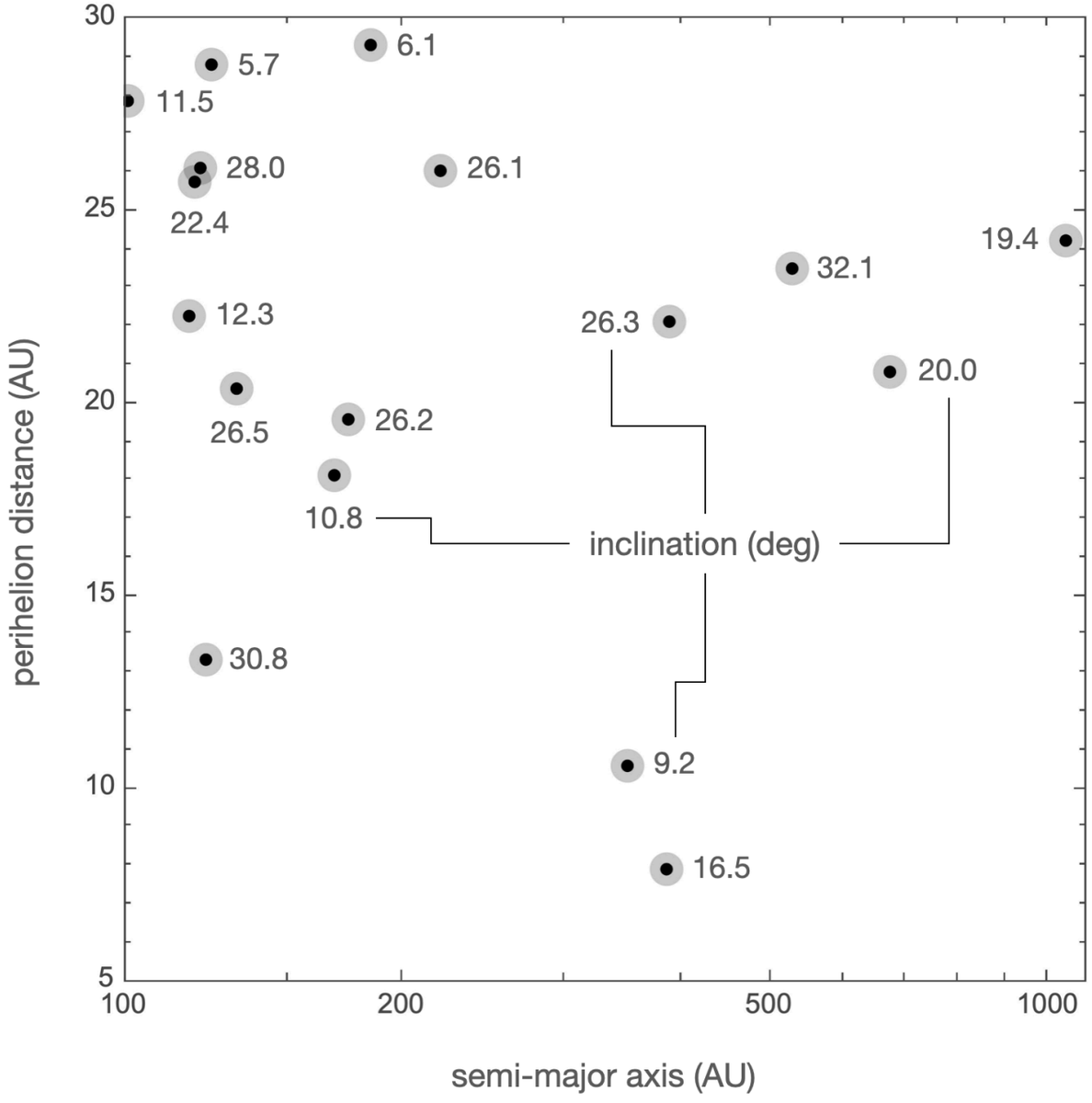
the simulations





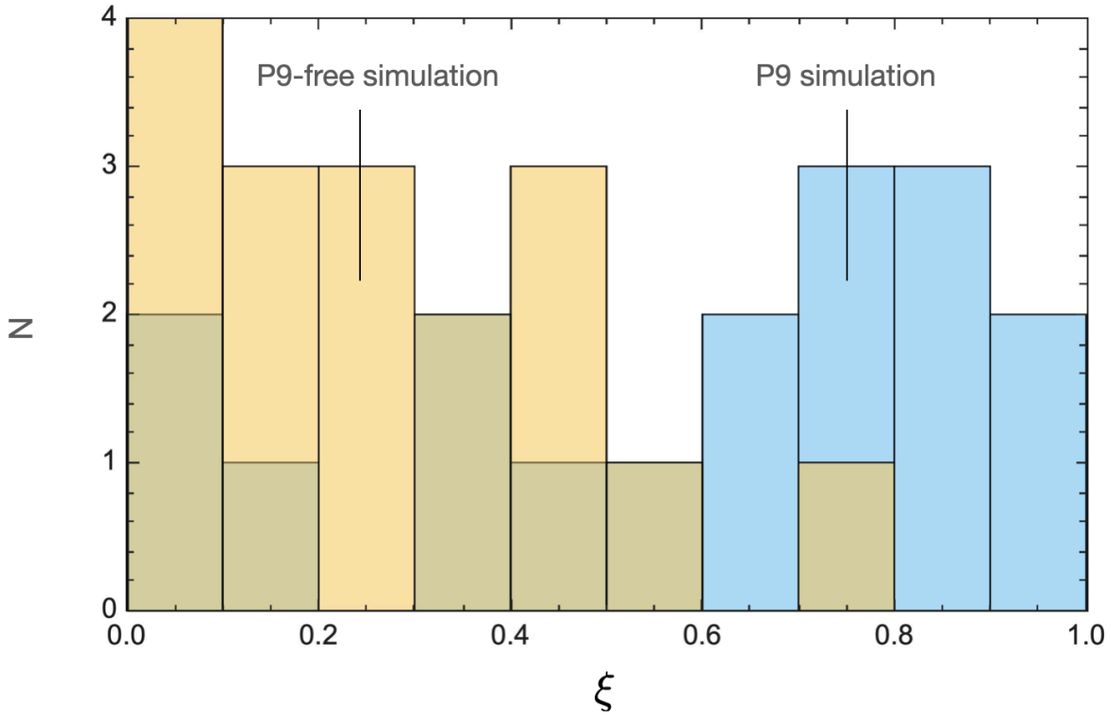
the data


top-down view
300 AU
Neptune's orbit
Y (AU)
X (AU)

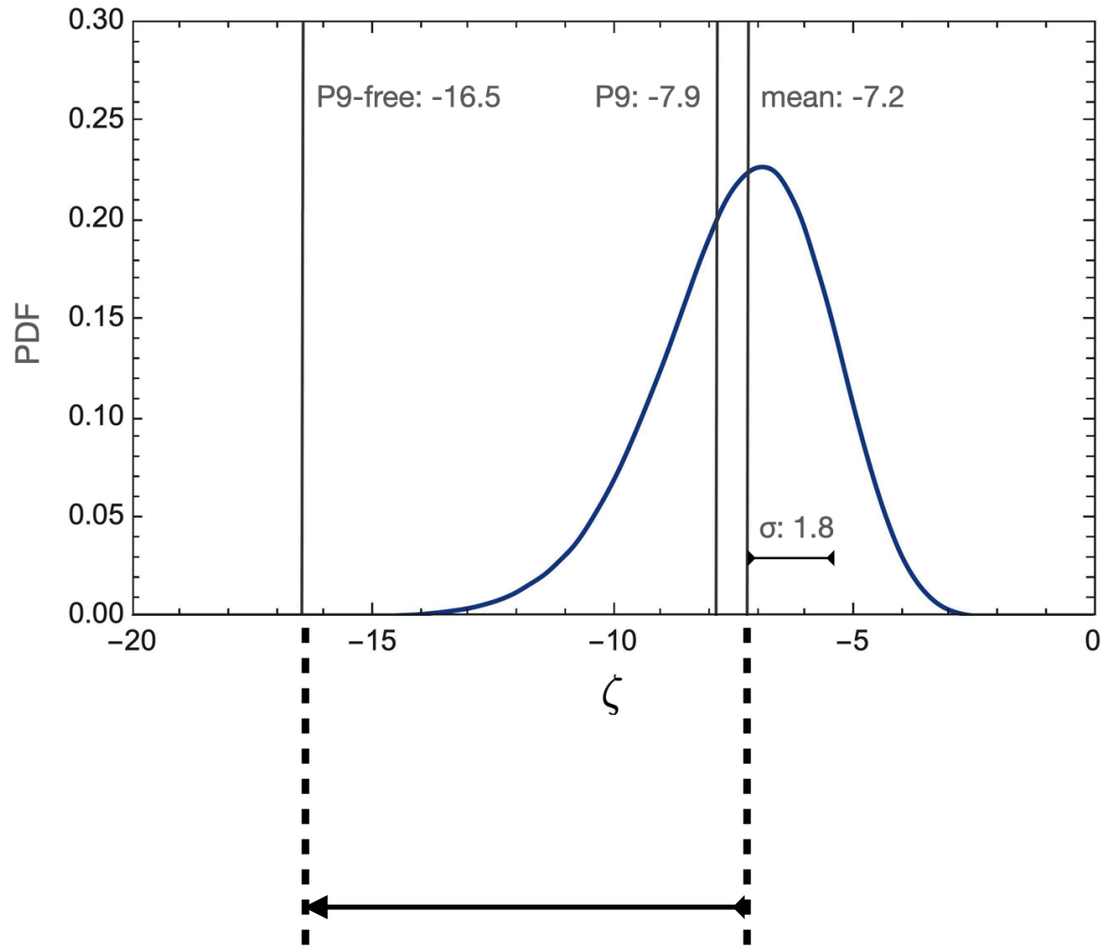

30
25
20
15
10
5
100
200
500
1000
perihelion distance (AU)
semi-major axis (AU)
inclination (deg)
6.1
5.7
11.5
28.0
26.1
22.4
19.4
32.1
12.3
26.3
20.0
26.5
26.2
10.8
30.8
9.2
16.5

accounting for the bias

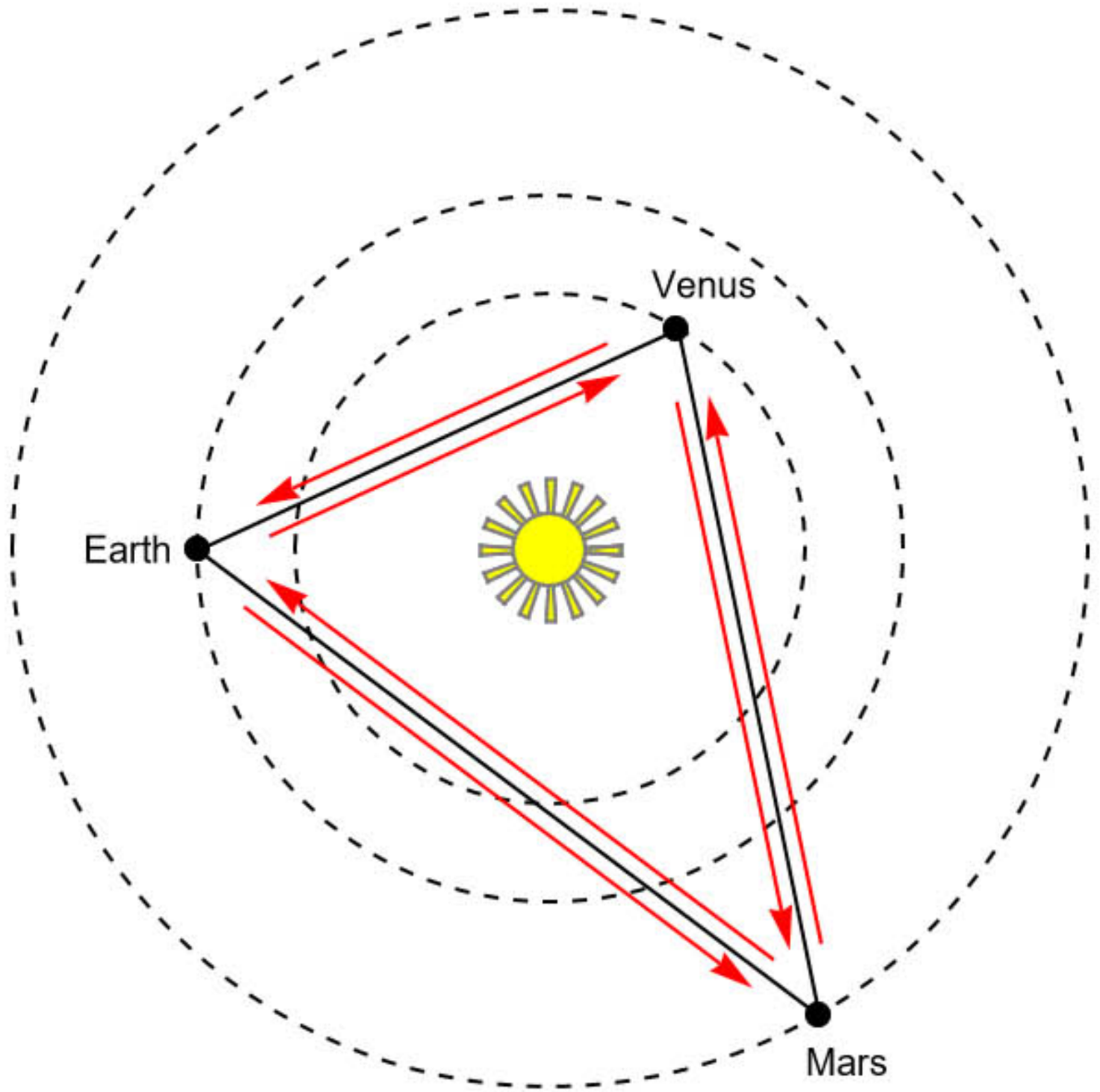
1. for each *discovery* distance of real objects, construct a theoretical perihelion distribution from the simulation
2. compute $\xi_j = \mathrm{CDF}_{r_j}(q_j)$. If model perfectly matches the data, this quantity should be uniformly distributed
3. quantify discrepancy via the statistic: $\zeta = \Pi_j^{N_{\mathrm{obj}}} \log(\xi_j)$ (approaches a Gaussian if \xi is uniform)



P9 KS-test $p = 0.4100$
P9-free KS-test $p = 0.0034$



P9-free: 5+ sigma away


Venus
Earth
Mars

All bets on gravity…..

A&A 640, A6 (2020)

New constraints on the location of P9 obtained with the INPOP19a planetary ephemeris

A. Fienga[1,2], A. Di Ruscio[3,1], L. Bernus[2,1], P. Deram[1], D. Durante[3], J. Laskar[2] and L. Iess[3]

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